The Estimation of Ground and Volume Polarimetry by Means of Pol-InSAR Measurements:

A Model-based Performance Analysis

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Ground-to-Volume Separation

Single Baseline
- Estimation of volume coherence is ambiguous
- Possible regularization:
  - Ground signal with minimal entropy (i.e. polarimetrically 2D ground)

Dual Baseline
- Assumption of the volume vertical reflectivity (i.e. profile)
  - Regularization = Choice in parametrization of profile

How good is the reconstruction of the volume / ground?

Approach: Inversions on forward simulations:
- Random volume over ground (RVoG) + Speckle noise
Forward Simulation

Polarimetry

Ground

- Dihedral
  \[ T_D = \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

- X-Bragg
  \[ T_{XB} = \begin{bmatrix} a & b \frac{\sin 2\beta}{2\beta} & 0 \\ b^* \frac{\sin 2\beta}{2\beta} & c \frac{1}{2} \left(1 + \frac{\sin 4\beta}{4\beta}\right) & 0 \\ 0 & 0 & c \frac{1}{2} \left(1 - \frac{\sin 4\beta}{4\beta}\right) \end{bmatrix} \]

Volume

- Distributed spheroids
  \[ T_V = \begin{bmatrix} 2 + \frac{4}{3} X + \frac{4}{15} X^2 & \frac{\sin 2\psi}{2\psi} \left(\frac{2}{3} X + \frac{4}{15} X^2\right) & 0 \\ \frac{\sin 2\psi}{2\psi} \left(\frac{2}{3} X + \frac{4}{15} X^2\right) & \frac{2}{15} X^2 \left(1 + \frac{\sin 4\psi}{4\psi}\right) & 0 \\ 0 & 0 & \frac{2}{15} X^2 \left(1 - \frac{\sin 4\psi}{4\psi}\right) \end{bmatrix} \]

\[ X = A_p - 1 \]
**Forward Simulation**

**Interferometry**
Random volume over ground (RVoG)

**Ground**

\[ |\gamma_g| = 1 \]

**Volume**

\[ \gamma^l_w = \frac{\int F(z) e^{i\kappa_l z} dz}{\int F(z) dz} \]

**Pol-InSAR**
double baseline (2 x single pass)

\[
T_8 \xrightarrow{\text{whitening}} \begin{bmatrix}
I & \Pi_{12} & 0 & 0 \\
\Pi^H_{12} & I & 0 & 0 \\
0 & 0 & I & \Pi_{34} \\
0 & 0 & \Pi^H_{34} & I \\
\end{bmatrix}
\]

\[ \Pi_l = \gamma_g T_{g,w} + \gamma^l_v T_{v,w} \]

**Scenarios**

I  
II  
III  

- exponential
- gaussian
- double layered

**Speckle**
complex Gaussian random vector with zero mean

\[ \tilde{U} = \mathcal{N}(0, I) \]

\[ T_{8,\text{noise}} = T_8^{\frac{1}{2}} \tilde{U}^H \tilde{U} T_8^{\frac{1}{2}} \]
Forward Simulation

Some examples:

1. Changing ground-to-volume ratio $\mu$

$N_{\text{looks}} = 100$

$k_z = 0.8$

$\mu_d = 1$

$\beta = 1$

$\epsilon = 10$

prof = ‘exp’

$h = 3$

$\sigma = 0.5$
**Inversion**

**Single baseline:**
1. Whitening
2. Estimation of the Center-of-Mass of coherence region
3. Shift along the line until ground coherency matrix has minimal entropy
4. De-whitening

\[
T_{g,w} = H \left\{ \frac{\Pi - \gamma_g I}{\gamma_g - \gamma_v} \right\}
\]

**Dual (multi) baseline:**
1. Whitening
2. Estimation of profile parameters by means of numerical minimization of

\[
\min_{\hat{h}, \sigma} \sum_j \left| \Pi_j - \gamma_v^j T_v - \gamma_g^j T_g \right|^2
\]

with

\[
T_{g,w} = \frac{1}{N} \sum_j H \left\{ \frac{\Pi_j - \gamma_g^j I}{\gamma_g^j - \gamma_v^j} \right\}, \quad T_{v,w} = I - T_{g,w} \quad \text{and} \quad \gamma_v^l = \frac{\int F(h, \sigma, z) e^{i\kappa z} z \, dz}{\int F(h, \sigma, z) \, dz}
\]

3. Reconstruction of the ground coherency matrix
4. De-whitening
Inversion

Baseline1: $k_z = 0.00$
Baseline2: $k_z = 0.80$

Profiles

true
single baseline
double baseline
Assessment of the Inversion Performance

• Relative error for 1000 runs:

$$\delta T_g = \frac{|T_g - T_{g,\text{model}}|}{|T_{g,\text{model}}|}$$

• Investigate distribution

• Compare with modeled variation by changing soil moisture:

$$\delta T_{g,\text{model}} = \frac{|T_g(m_{v\text{max}}^2) - T(m_{v\text{min}})|}{|T(m_{v\text{max}}) + T(m_{v\text{min}})|}$$
Results – Single vs Double Baseline

Distribution of relative error on ground polarimetry

\[ \delta T_g = \frac{|T_g - T_{g, model}|}{|T_g, model|} \]
Results – Particle Anisotropy and Orientation

\[ \delta T_g = \frac{|T_g - T_{g,\text{model}}|}{|T_{g,\text{model}}|} \]

Scenario I estimated with
- \( A_p = 0.10 \)
- \( A_p = 0.32 \)
- \( A_p = 1.00 \)
- \( A_p = 3.16 \)
- \( A_p = 10.00 \)

Scenario II estimated with
- \( D_{\phi} = 0.00 \) [rad]
- \( D_{\phi} = 0.39 \) [rad]
- \( D_{\phi} = 0.79 \) [rad]
- \( D_{\phi} = 1.18 \) [rad]
- \( D_{\phi} = 1.57 \) [rad]
Results – Baseline Dependency

Scenario II

Exponential Profile

Gaussian Profile

relative error of $T_g$ (mean over 100 simulations)
Results – Baseline Dependency

Scenario I

Exponential Profile

Scenario II

Gaussian Profile

Scenario V

Exponential Profile

Gaussian Profile
Comparing Results with Soil Moisture Variations

Scenario I

Inversion results (200 looks)

- double baseline - exp
- double baseline - gauss
- single baseline

\[
\frac{|T(m_{v_{\text{max}}}) - T(m_{v_{\text{min}}})|}{|T(m_{v_{\text{max}}}) + T(m_{v_{\text{min}}})|}
\]

Volumetric moisture content \( m_v \) [g/cm\(^3\)]

Volumetric moisture content \( m_v \) [g/cm\(^3\)]
Comparing Results with Soil Moisture Variations

Scenario II

Inversion results (200 looks)

- blue: double baseline - exp
- orange: double baseline - gauss
- green: single baseline
Comparing Results with Soil Moisture Variations

Scenario I

Inversion results (200 looks)

- double baseline - exp
- double baseline - gauss
- single baseline
Conclusions

Model based separation & reconstruction of the polarimetric ground and volume scattering contributions from Pol-InSAR data can be achieved with an accuracy that may allow a meaningful estimation of surface parameters.

**Dual-baseline** reduces the error about 5% compared to single baseline.

**Parametrization** with Gaussian profile appear to be more stable than exponential profiles.

However, high dependency on the **baseline combinations**.

When changing **polarimetry of volume scatter**, the particle orientation has no influence while the particle shape has minor influence: Best results achieved with spheres.

**Comparison with change in soil moisture**: using 200 looks, the Inversion is sensitive to capture ±5% change in soil moisture (around soil moisture = 20%)
Grazie mille!
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Forward Simulation

Some examples:

2. Changing X-bragg roughness $\beta$

$N_{\text{looks}} = 100$
$k_z = 0.8$

$\mu_d = 1$
$\mu = 2$
$\varepsilon = 10$

prof = ‘exp’
$h = 3$
$\sigma = 0.5$
Results – Multilooking

\[
\delta T_g = \frac{|T_g - T_{g,\text{model}}|}{|T_{g,\text{model}}|}
\]

Scenario I estimated with

\[
\delta T_g = \frac{|T_g - T_{g,\text{model}}|}{|T_{g,\text{model}}|}
\]

Scenario II estimated with

\[
\delta T_g = \frac{|T_g - T_{g,\text{model}}|}{|T_{g,\text{model}}|}
\]
Conclusions

Model based separation & reconstruction of the polarimetric ground and volume scattering contributions from Pol-InSAR data can be achieved with an accuracy that may allow a meaningful estimation of surface and volume parameters.

**Dual- v.s Single-baseline:** The availability of Dual-baseline data reduces the error of the estimated ground signal about 5% compared to single baseline.

**Parametrization of the profiles:** Gaussian profiles appear to be more stable than exponential profiles. However: there is a higher dependency on the baseline combinations.

**Changing Polarimetry of volume scatter:** The particle orientation has no influence while the article shape has minor influence: Best results achieved with spheres.

**Comparison with change in soil moisture:** using 200 looks, the Inversion is sensitive to capture ±5% change in soil moisture (around soil moisture = 20%)
Results – Single vs Double Baseline

Scenario I

Scenario II

Scenario III

All Scenarios

$\delta T_g = \frac{|T_g - T_{g,model}|}{T_{g,model}}$
How good is the volume-to-ground separation?

❖...under different scenarios
  • Vertical extinction and reflectivity profiles
  • Ground-to-volume ratios

❖...with different observational settings
  • Single or double baseline

❖...under varying regularization
  • Assuming different vertical profiles

Approach: Forward Simulation

Random volume over ground RVoG
  + Speckle noise
Conclusions

• **Double v.s single baseline:**
  • Double baseline reduces error of estimated ground signal about 5% compared to single baseline

• **Parametrization of the profiles:**
  • Gaussian appears to be more stable than exponential
  • However: high dependency on baseline combinations

• **Ground-to-Volume ratio** affects the estimation of ground signal more than volume signal

• **Changing Polarimetry of volume scatter**
  • Particle orientation has no influence
  • Particle shape has minor influence: best results when shape is a sphere

• **Comparison with change in soil moisture:**
  • With 200 looks, the Inversion is sensitive to capture ±5% change in soil moisture (around soil moisture = 20%)
Forward Simulation

Some examples:

3. Changing number of looks $N_{\text{looks}}$

$k_z = 0.8$

$\mu_d = 1$
$\mu = 0.5$
$\beta = 0.5$
$\epsilon = 10$

prof = ‘exp‘
$h = 3$
$\sigma = 0.5$
Results – Multilooking

Distribution of relative error on ground polarimetry

\[ \delta T_g = \frac{|T_g - T_{g, model}|}{|T_g, model|} \]
Results – Ground vs Volume estimation

\[ \delta T_{g/v} = \frac{|T_{g/v} - T_{g/v,\text{model}}|}{|T_{g/v,\text{model}}|} \]