Quantitative Evaluation of Ionospheric Distortions on Spaceborne L-band SAR under Faraday rotation

Jun Su Kim and Kostas Papathanassiou
Microwaves and Radar Institute, German Aerospace Center
FR and SAR Polarimetry

\[ 0 = Y \left( \begin{array}{c} \frac{1}{f_R} \\ \delta_{RH} \\ \delta_{RV} \end{array} \right) \left( \begin{array}{cc} \cos \Omega & \sin \Omega \\ -\sin \Omega & \cos \Omega \end{array} \right) \left( \begin{array}{cc} S_{HH} \\ S_{HV} \end{array} \right) \left( \begin{array}{cc} \cos \Omega & \sin \Omega \\ -\sin \Omega & \cos \Omega \end{array} \right) \left( \begin{array}{c} \frac{1}{f_T} \\ \delta_{TH} \end{array} \right) \]

- **TEC Estimation**

1° FR corresponds to 0.336 TECU at L-band 
(when \( \vec{B} \cdot \hat{\kappa} = 40 \text{ K nT} \)) 2.88 TECU at P-band

- **Vertical Structure**

1° FR contributes to \( \pm 3.55 \times 10^{11} \text{ el./m}^3 \)

- **Ionospheric Phases**

\[ \Omega = \zeta \frac{e \vec{B} \cdot \hat{\kappa}}{cmf^2} TEC \]

\[ \Omega = \frac{e \vec{B} \cdot \hat{\kappa}}{4\pi mf} \phi \]

1° FR corresponds to 6.8 phase cycles at L-band
(when \( \vec{B} \cdot \hat{\kappa} = 40 \text{ K nT} \)) 2.3 phase cycles at P-band
Polarimetric Distortion from FR

• There are three main TEC cycles:
  • 1. Solar cycle with ca. 11-year of period. (but not exactly periodic)
  • 2. Annual (seasonal) cycle
  • 3. Daily cycles,

Additionally there is a day-to-day variation (due to neutral atmosphere, cosmic and solar particle precipitation, the fluctuation of geomagnetic field, and so on)

• The FR variation is analysed using
  • IGN (International GNSS Network) TEC map: 5° × 2.5° × 2 hours, since 1998 on.
  • SAR imaging geometry (altitude, inclination, off-nadir angle and look direction)
  • Geomagnetic field model

• The effects on FR on SAR data at mid-latitudes is analysed for a L-band SAR over Kaufbeuern, Germany.

• Dual-pol FR estimation and data correction is analysed.
Orbit and Ionospheric Piercing Point

- A sentinel-like orbit is studied (175 cycles/12 days, incl.: 98.2°, alt.: 698.56 km)
- Ionospheric altitude of 300 km is assumed, for the calculation of IPP and for the conversion from IGN TEC to FR.
- Right looking geometry is applied.

Black: ground position
Green: ionospheric piercing points (off nadir angel 30°)

Right looking geometry leads to a wider hole at the south pole
Calculation of TEC and FR

- Distribution of TEC map and orbit, and estimated TEC and FR.

**Dawn**

1998-06-02 00:00 UTC

**Dusk**

This process is repeated for 175 revolutions in 12 days. This cycle repeats 617 times through 1998-2018.
• Distribution of TEC per each revisit cycles (Ascending Dawn/Descending Dawn)

- Blue to red curves indicate 10 to 100 percentiles (10 steps) of TEC distributions.
- Percentiles are calculated in the samples in 12 days.
- 2-Periods: Annual and solar cycle.
- Evening time (≈18 LT) show around twice stronger TEC at 50th percentile. (10 and 20 TECU)
- Large deviation of TEC (because it is average of wide latitudes)
Geographical TEC distribution

- TEC of dawn and dusk times (LTAN at dawn) Averaged through 1998-2018

- Averaged TEC on morning time (top left) and evening time (bottom right).

- TEC difference is observed only along low latitudes.

- Difference between morning and evening is small at high latitudes.

- Evident equatorial anomaly
FR analysis over Kaufbeuern, Germany

- Orbits imaging Kaufbeuern at different off-nadir angles. (Asc./Des. orbits)

- FRs under given imaging geometry retrieved from IGN TEC map
At the co-pol channels, the FR effect is similar regardless of scattering type.
At mid-latitudes, less than 0.1 dB of bias in power is expected for 80% of conditions.
The sample-to-sample effect (relevant for dynamics) can and is significantly higher
Cross-pol power under FR (Asc. Dawn 38.0°)

- Effect of FR is strongly dependent on the scattering type.
- The less the contrast between the channels, the less the effect of FR. (e.g. Forest)
- HV and VH channels behave different.
FR Estimation in Dual-pol Observation Space

\[
\begin{pmatrix}
M_{hh} & M_{hv} \\
M_{vh} & M_{vv}
\end{pmatrix} = \begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix} \begin{pmatrix}
S_{hh} & S_{xx} \\
S_{xx} & S_{vv}
\end{pmatrix} \begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix}
\]

\[
= \begin{pmatrix}
S_{hh} \cos^2 \Omega - S_{vv} \sin^2 \Omega \\
-(S_{hh} + S_{vv}) \cos \Omega \sin \Omega + S_{xx}
\end{pmatrix}
\begin{pmatrix}
(S_{hh} + S_{vv}) \cos \Omega \sin \Omega + S_{xx} \\
-S_{hh} \sin^2 \Omega + S_{vv} \cos^2 \Omega
\end{pmatrix}
\]

H-transmit Dual-pol

- Dual-pol FR estimator (proposed by Jeremmy Nicoll’s – personal communication):

\[
\Omega = \arg\langle d_m d_p^* \rangle / 4,
\]

where \( d_m = M_{hh} + iM_{hv} \) and \( d_p = M_{hh} - iM_{hv} \)

- Comparison with quad-pol estimator

\[
\Omega = \arg\langle d_m d_p^* \rangle / 4,
\]

where \( d_m = (M_{hh} + M_{vv}) + i(M_{hv} - M_{vh}) \) and \( d_p = (M_{hh} + M_{vv}) - i(M_{hv} - M_{vh}) \)

- Dual-pol estimator is simply drop-off version of V-transmits from quad-pol estimator.

- Subtraction between off-diagonal components is the key part of FR estimation because it measures the violation of reciprocity. Dual-pol cannot see it.
Performance under controlled scattering matrix

Diagonal: \[ \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix}, \alpha \in \mathbb{C} \]

Off-diag.: \[ \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}, \beta \in \mathbb{C} \]

• The main obstacle of dual-pol FR estimation is the lack of reference for reciprocity test. (incomplete off-diag.)

• The higher the FR is, the more sensitivity on the baseline scattering mechanism.

• The real SAR data (underlying contours) indicates strong spatial averaging of dual-pol FR can approximate the measurement to the true FR.
Comparison of quad- and dual-pol FR Estimates

ALOS PALSAR 2007/03/18 02:33:20 – 02:37:44  Northeast Brazil

Quad-pol FR
Dual-pol FR

ALOS PALSAR 2007/05/03 02:33:20 – 02:37:44  Northeast Brazil

Quad-pol FR
Dual-pol FR
Correction Applied on Dual-pol Data

• Two assumptions of scattering matrix reconstruction
  • 1. reflection symmetry after correction ($S'_{hv} = S'_{vh}$)
  • 2. $M_{hh} = M_{vv}$

\[
\begin{pmatrix}
S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{pmatrix} =
\begin{pmatrix}
\cos \Omega & - \sin \Omega \\
\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
M_{hh} & M_{hv} \\
M_{vh} & M_{vv}
\end{pmatrix}
\begin{pmatrix}
\cos \Omega & - \sin \Omega \\
\sin \Omega & \cos \Omega
\end{pmatrix}
\]

Corrected matrix

H-transmit Dual-pol

• From condition 1 and 2, the not measured $M_{hv}$ is assumed to be given:

\[
M_{hv} = 2M_{hh} \tan 2\Omega + M_{vh}
\]

• The original scattering matrix elements are then obtained as

\[
S_{hh} = \frac{M_{hh}}{\cos 2\Omega} \quad \text{and} \quad S_{vh} = M_{hh} \tan 2\Omega + M_{vh}
\]

• The Faraday rotation $\Omega$ is assumed to be known.
Dual-pol Correction Performance (hh) $\sigma = 0.16^\circ$

- At the co-pol channels, the FR effect is similar regardless of scattering type.
- At mid-latitudes, less than 0.1 dB of bias in power is expected for 80% of conditions.
Dual-pol Correction Performance (vh) $\sigma = 0.16^\circ$

- Effect of FR is strongly dependent on the scattering type.
- The less the contrast between the channels, the less the effect of FR. (e.g. Forest)
- HV and VH channels behave differently.
Dual-pol Correction Performance (hh) $\sigma = 1^\circ$
Dual-pol Correction Performance (vh) $\sigma = 1^\circ$
Conclusion

• Realistic FR estimates are calculated based on IGN TEC map, obs. geometry and $\vec{B}$.
• FR at Dusk time is about 50% stronger than at Dawn time. (06/18 comparison)
• Enhancement of FR at Dusk time is prominent only around equator.

• FR estimates for Kaufbeuern, Germany, shows weak dependency on incidence angle.
• Shallow incidence angles correspond to larger slant TECs.
• Mean FR is expected to be less than 3° (50% expectation) at Dawn time, and less than 4° (50% expectation) at Dusk time.
• Co-pol channels suffer less than 0.1 dB of decrease for 80% of the expected FRs levels.
• The behaviour of the Cross-pol channels is more dynamic: The distortion is minimised with degreasing contrast between the channels implying a strong dependency on scattering type.
• The sample-to-sample effect (relevant for dynamics) is significantly higher.

• The main limitation of Dual-pol FR estimators is the incompleteness of the reciprocity pair.
• Assuming an ideal Dual-pol calibration performance Dual-pol estimators can potentially overcome this by using enormously large multi-looking with the help of azimuth symmetry ($\langle S_{hh} \cdot S_{xx}^T \rangle = 0$).
• Dual-pol performance degrades with increasing FR (FR>10° cases).
Quantitative Evaluation of Ionospheric Distortions on Spaceborne L-band SAR under Faraday rotation

Jun Su Kim and Kostas Papathanassiou
Microwaves and Radar Institute, German Aerospace Center